Split-Tee Power Divider

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***Abstract*—A new type of power divider is described which provides two in-phase isolated outputs with a constant arbitrary power division over a wide bandwidth. The derivation of the formulas from which the unit may be deisgned and its performance predicted are given. The bandwidth characteristics of a two-third, one-third power divider are given along with experimental verification.**

Introduction

A LARGE NUMBER of different types of power dividers, with and without isolation between output ports, are used for various applications. Certain symmetrical power dividers, which provide isolation between output ports, either have quadrature outputs or, depending upon the input port, in-phase or out-of-phase outputs. Among the power dividers which provide quadrature outputs are the multiple branch waveguide coupler,[1] the short-slot hybrid,[2] and the contradirectional coupler.[3,4] Power dividers which provide in-phase or out-of-phase components consist of the magic T,[5] the N-way power divider,[6] and the hybrid-ring directional coupler.[7] In the design of a microwave distribution network, a power divider, providing two equal phase outputs with unequal power division, is often required. The hybrid-ring directional coupler may be used for this purpose. However, the ratio of output power changes by about 1.5 dB over a 1.5 to 1 band, and the phase difference of the two outputs varies by about $\pm 4°$ over a 1.15 to 1 band. Another method of obtaining the desired power split with equal phase outputs is the use of a broadband quadrature coupler and 90° phase shifter.[8] This relatively complex method requires close manufacturing tolerances and hence is undesirable.

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The split-tee power divider, when constructed in strip line, is a simple compact, broadband device. It provides two isolated, equal phase, unequal amplitude, outputs with a good match at each port. This power divider is similar to the N-way power divider which provides N equiphase, equiamplitude, isolated ports. In fact, the split-tee power divider may be developed from the N-way power divider as follows: connect n of the output ports together to form one port and the remaining $N-n$ output ports together to form the other port, connect quarter-wave transformers to the two resulting output ports to adjust their impedance level, and a power divider with two equiphase outputs and power ratio, n to $N-n$, has been derived.

Split-Tee Power Divider

A strip line version of the split-tee power divider is shown in Fig. 1. The isolation resistor, shown as a thin film resistor, can be replaced by a quarter-watt carbon resistor at or below L-band frequencies. The detailed center conductor configuration with design equations is given in Fig. 2. These equations will be derived assuming that the cross-sectional dimensions of the strip line are very small compared to a wavelength so that discontinuities caused by corners and junctions are negligible. Consider the unequal power divider shown in Fig. 3. The power divider is designed so that when fed from port 1, a match is seen; the power out of port 3 is K^2 times that out of port 2; and the voltage between arm 2 and ground is equal to that between arm 3 and ground when measured at equal distances from port 1. To satisfy the second and third of the above conditions, all impedances in arm 2 must be K^2 times the corresponding impedances in arm 3. To see a match at port 1, it follows that

$$\frac{Z_{i2}Z_{i3}}{Z_{i2}+Z_{i3}} = \frac{K^2 Z_{i3}}{1+K^2} = Z_0$$

$$Z_{i3} = \frac{1+K^2}{K^2} Z_0 \tag{1}$$

where Z_{i2} and Z_{i3} are the input impedances looking into arms 2 and 3 from port 1.

The output impedances R_2 and R_3 are chosen to be

$$R_2 = KZ_o \qquad R_3 = \frac{Z_o}{K} \tag{2}$$

As will be shown, this choice causes the output transformers to have identical phase-transfer characteristics.

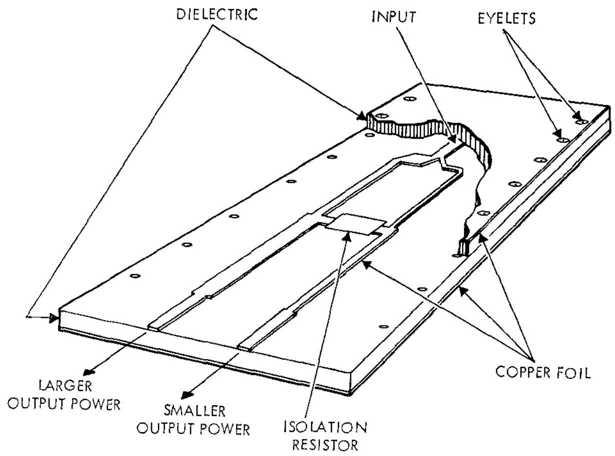

Fig. 1. Strip-line split-tee power divider.

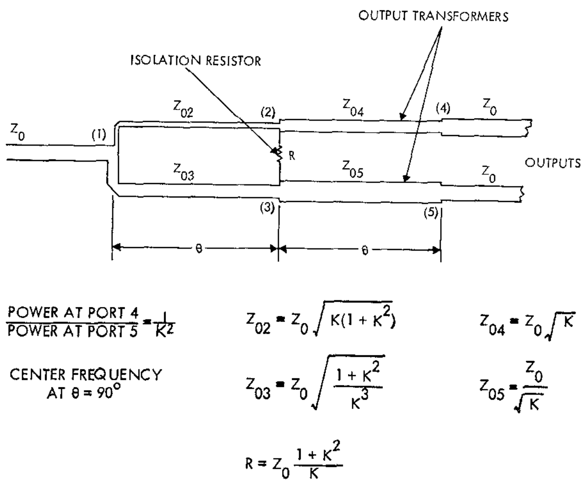

Fig. 2. Design equations for the split-tee power divider.

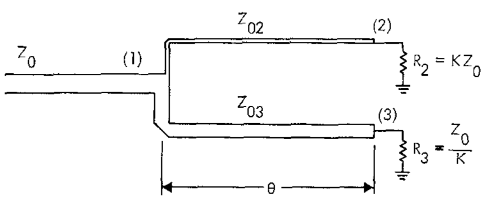

Fig. 3. Power divider with unequal output impedances.

Using (1) and (2), the characteristic impedances of the quarter-wave transformers in arms 2 and 3 are determined as

$$Z_{o2} = Z_o\sqrt{K(1 + K^2)} \qquad Z_{o3} = Z_o\sqrt{\frac{1 + K^2}{K^3}} \tag{3}$$

With this design, the voltages at port 2 and port 3 are equal. Hence, a resistor may be placed between these two ports without causing any power dissipation. If the power divider is fed from port 2 or port 3, energy will be dissipated in the resistor. Isolation between output ports and a good match seen looking in at any port is obtainable because of this resistor. Although the proper value of the isolation resistor has not been derived, it will be shown in the next section that the value given in Fig. 2 yields infinite isolation and a perfect match at the center frequency.

The design of a power divider with unequal output impedances has been accomplished. The function of the output transformers Z_{o4} and Z_{o5} is simply to transform the two unequal output impedances to Z_o. The proper values of Z_{o4} and Z_{o5} are given in Fig. 2.

Analysis of the Split-Tee Coupler with Unequal Output Impedances

The characteristics of a microwave network usually are best described by the scattering matrix. However, when the network has unequal impedance levels, the scattering matrix coefficients are not the simplest to use. Instead, the *unnormalized* voltage scattering matrix coefficients will be used to describe the network shown in Fig. 4. These coefficients are most clearly defined by the following matrix:

$$\begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} & T_{13} \\ T_{21} & T_{22} & T_{23} \\ T_{31} & T_{32} & T_{33} \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix} \tag{4}$$

where d_j ($j=1, 2, 3$) is the amplitude and phase of the voltage wave leaving the jth-port, and c_k ($k=1, 2, 3$) is the amplitude and phase of the voltage wave incident upon the kth-port. First the transfer coefficients T_{11}, T_{21}, and T_{31} will be determined. These coefficients are determined by inserting a unit voltage wave at port 1 ($c_1=1$), match terminating port 2 and port 3 ($c_2=c_3=0$), and determining the waves leaving ports 1, 2, and 3 (d_1, d_2, and d_3).

Since the network is driven at port 1, the voltage from arm 2 to ground is the same as that from arm 3 to ground at any distance from port 1. Hence, a short-circuiting line may be used to connect arms 2 and 3 together. The resulting network is shown in Fig. 5 where arms 2 and 3 have been connected in parallel as have the load resistors R_2 and R_3.

The equations which govern the network are

$$\begin{aligned} d_1 &= \Gamma_1 c_1 + (1 - \Gamma_1)c_a \\ d_a &= (1 + \Gamma_1)c_1 - \Gamma_1 c_a \\ c_a &= \Gamma_1 e^{-j2\theta} d_a \\ d_2 &= (1 + \Gamma_1)e^{-j\theta} d_a \end{aligned} \tag{5}$$

where

$$\Gamma_1 = \frac{\sqrt{K} - \sqrt{1 + K^2}}{\sqrt{K} + \sqrt{1 + K^2}} \tag{6}$$

Solving the preceding equations, we have

$$T_{11} = \left.\frac{d_1}{c_1}\right|_{c_2=c_3=0} = \Gamma_1\left[1 + \frac{(1 - \Gamma_1^2)e^{-j2\theta}}{1 + \Gamma_1^2 e^{-j2\theta}}\right]$$

$$T_{21} = T_{31} = \left.\frac{d_2}{c_1}\right|_{c_2=c_3=0} = \frac{(1 + \Gamma_1)^2 e^{-j\theta}}{1 + \Gamma_1^2 e^{-j2\theta}} \tag{7}$$

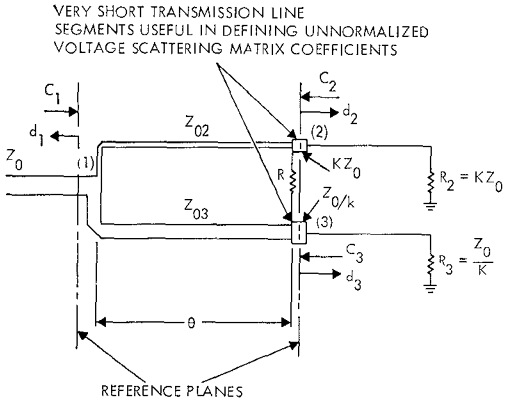

Fig. 4. Definition of unnormalized voltage scattering matrix coefficients.

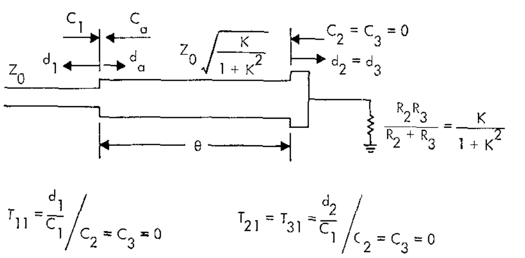

Fig. 5. Equivalent circuit for determining T_{11}, T_{21}, and T_{31}.

The values of T_{12} and T_{13} may be obtained using reciprocity. They are

$$T_{12} = \frac{T_{21}}{K} \qquad T_{13} = KT_{31} \tag{8}$$

The above coefficients were easily derived because, when driven from port 1, the power divider may be analyzed as a two-port network. The remaining coefficients (T_{23}, T_{32}, T_{22}, and T_{33}) will be determined by simultaneously exciting port 2 and port 3 in-phase (even excitation), and then exciting them out-of-phase (odd excitation), as illustrated in Fig. 6. The voltages and currents at port 2 and port 3 will be obtained for each set of excitations. By using superposition, the desired *unnormalized* voltage scattering matrix coefficients will then be obtained.

For the even excitation, the voltage generators are excited with $V_{2e} = V_{3e}$ and $V_{20} = V_{30} = 0$. Consider the two networks shown in Fig. 7(a). E_{2e}, I_{2e}, E_{3e}, and I_{3e} may be obtained by inspection as

$$I_{2e} = \frac{V_{2e}}{KZ_0 + Z_{2e}} \qquad E_{2e} = \frac{Z_{2e}V_{2e}}{KZ_0 + Z_{2e}}$$

$$Z_{2e} = Z_{02}\frac{Z_0(1 + K^2) + jZ_{02}\tan\theta}{Z_{02} + jZ_0(1 + K^2)\tan\theta}$$

$$I_{3e} = K^2 I_{2e} \qquad E_{3e} = E_{2e} \tag{9}$$

Since the impedance at every point along arm 2 is K^2 times the impedance at the corresponding point along arm 3, the voltage distribution in both networks is identical and the current in the lower impedance network is K^2 times that of the higher impedance network. Because the voltages along both arms are identical, the two networks in Fig. 7(a) may be joined together to form the network given in Fig. 6 without changing the voltage or current distribution. Hence, for the even excitation, the voltages and currents are given by (9).

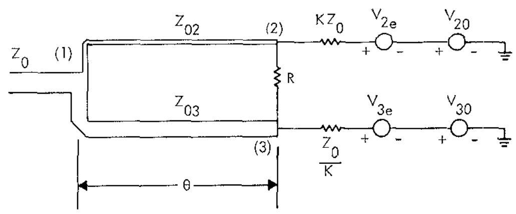

Fig. 6. Even and odd excitations.

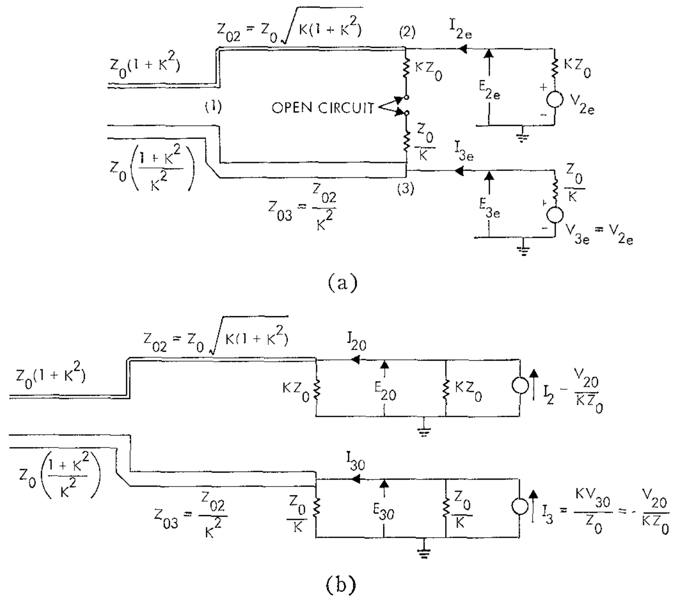

Fig. 7. (a) Equivalent circuit for even excitation. Note that the isolation resistor which has the value $R = Z_0/K + KZ_0$ has been split into two parts. (b) Equivalent circuit for odd excitation.

To produce the odd excitation, the voltage generators of Fig. 6 are excited so that $V_{2e} = V_{3e} = 0$ and $V_{20} = -K^2 V_{30}$. Consider the two networks given in Fig. 7(b). Both networks are driven by equal magnitude current sources. The voltages and currents are determined from inspection to be

$$E_{20} = \frac{V_{20}}{1 + KY_{02}Z_0} \qquad I_{20} = \frac{V_{20}Y_{02}}{1 + KY_{02}Z_0}$$

$$Y_{02} = \frac{1}{KZ_0} + \frac{1}{jZ_0\sqrt{K(1 + K^2)}\tan\theta}$$

$$E_{30} = -\frac{E_{20}}{K^2} \qquad I_{30} = -I_{20} \tag{10}$$

Since the admittance at every point along line 2 is $1/K^2$ times the admittance at the corresponding point along line 3, the currents will distribute themselves identically in both networks. Each set of ground leads, one connected to port 1 and the other to the isolation resistor,

θ	T_{11}	$T_{21}=T_{31}=\sqrt{2}T_{12}=\frac{T_{13}}{\sqrt{2}}$	T_{33}	$T_{23}=2T_{32}$	T_{22}	Isolation (dB)
70°	$-0.047+j.121$	$0.682\underline{/-68.7°}$	$0.025-j.040$	$0.043-j.163$	$0.005+j.042$	18.5
80°	$-0.012+j.065$	$0.685\underline{/-79.3°}$	$0.007-j.023$	$0.009-j.084$	$0.002+j.018$	24.3
90°	0	$0.686\underline{/-90°}$	0	0	0	∞
100°	$-0.012-j.065$	$0.685\underline{/-100.6°}$	$0.007+j.023$	$0.009+j.084$	$0.000-j.018$	24.3
110°	$-0.047-j.121$	$0.682\underline{/-111.3°}$	$0.025+j.040$	$0.043+j.163$	$0.005+j.042$	18.5

carry currents equal in magnitude and opposite in phase. Hence, the ground leads may be removed and the portions of the two circuits of Fig. 7(b) connected together to form the network of Fig. 6 without changing the current distribution. Therefore, (10) applies for the odd excitation of the power divider.

Using (9) and (10), the desired voltage transfer coefficients may be obtained by superposition of odd and even excitations. For instance, if $V_{20}=-V_{2e}$, port 2 is not excited and port 3 is excited with the voltage $V=V_{3e}+V_{30}=V_{2e}-V_{20}/K^2=V_{2e}(1+1/K^2)$. If a short transmission line of characteristic impedance Z_0/K is inserted between the generator resistance Z_0/K and port 3 (see Figs. 6 and 7), the incident voltage wave will be $V_{2e}(1+K^2)/2K^2$. The total voltage across port 3 is

$$V_{T3} = (1+T_{33})V_{\text{inc}} = (1+T_{33})\,\frac{V_{2e}(1+K^2)}{2K^2}$$

$$V_{T3} = E_{3e}+E_{30} = \frac{Z_{2e}V_{2e}}{KZ_0+Z_{2e}} + \frac{V_{2e}}{K^2(1+KY_{02}Z_0)}$$

$$T_{33} = \frac{2K^2}{1+K^2}\left[\frac{Z_{2e}}{KZ_0+Z_{2e}} + \frac{1}{K^2(1+KY_{02}Z_0)}\right] - 1 \quad (11)$$

Using the same set of excitations,

$$T_{23} = \frac{2K^2V_{T2}}{(1+K^2)V_{2e}} = \frac{2K^2(E_{2e}+E_{20})}{(1+K^2)V_{2e}}$$

$$T_{23} = \frac{2K^2}{1+K^2}\left[\frac{Z_{2e}}{KZ_0+Z_{2e}} - \frac{1}{1+KY_{02}Z_0}\right] \quad (12)$$

To determine T_{22} and T_{32}, set $V_{30}=-V_{3e}$ so that port 3 is not excited and port 2 is excited with the voltage $V=V_{2e}+V_{20}=V_{2e}-K^2V_{30}=V_{2e}(1+K^2)$. Inserting a short transmission of characteristic impedance KZ_0 between the the generator impedance KZ_0 and port 2, the incident voltage wave is $\frac{1}{2}V_{2e}(1+K^2)$. The total voltage at port 2 is

$$V_{T2} = (1+T_{22})V_{\text{inc}} = \tfrac{1}{2}(1+K^2)(1+T_{22})V_{2e}$$

$$V_{T2} = E_{2e}+E_{20} = \frac{Z_{2e}V_{2e}}{KZ_0+Z_{2e}} + \frac{V_{20}}{1+KY_{02}Z_0}$$

$$= \frac{Z_{2e}V_{2e}}{KZ_0+Z_{2e}} + \frac{K^2V_{2e}}{1+KY_{02}Z_0}$$

$$T_{22} = \frac{2}{1+K^2}\left[\frac{Z_{2e}}{KZ_0+Z_{2e}} + \frac{K^2}{1+KY_{02}Z_0}\right] - 1 \quad (13)$$

With the same excitation,

$$T_{32} = \frac{2V_{T3}}{(1+K^2)V_{2e}} = \frac{2(E_{3e}+E_{30})}{(1+K^2)V_{2e}}$$

$$T_{32} = \frac{2}{1+K^2}\left[\frac{Z_{2e}}{KZ_0+Z_{2e}} - \frac{1}{1+KY_{02}Z_0}\right] \quad (14)$$

The complete characteristics of the split-tee power divider with unequal output impedances is described by (7), (8), (11)–(14). The isolation between output ports is the ratio of the power out of port 3 to that incident upon port 2 and is

$$\text{Isolation} = 10\log K^2\,|T_{32}|^2 = 10\log\frac{|T_{23}|^2}{K^2} \quad (15)$$

In order to estimate the broadband characteristics of the power divider, a unit with $K^2=2$ was designed. The characteristics of this unit were computed from the derived equations and are presented in tabular form. From the chart, it is seen that over the considered range of θ, the split-tee power divider has good VSWR (voltage standing-wave ratio), isolation and perfect phase identity and output ratio constancy.

Split-Tee Power Divider with Equal Output Impedances

The power divider with unequal output impedances has been discussed in detail. It is apparent that if multiple section quarter-wave transformers are used at each output, a power divider with excellent amplitude and phase tracking characteristics and with equal output impedances is obtainable. However, single section output transformers are adequate for most applications.

Single section output transformers permit good amplitude and phase tracking because the phase of the transmission coefficient of a quarter-wave transformer is dependent upon the transformer ratio and not upon whether it is a step up or step down transformer. (Note that in (7) the phase of T_{12} is the same for Γ_1 positive or negative.) From Fig. 2, it is seen that both output transformers have the same transformer ratio. Since the incoming waves at port 2 and port 3 are in phase except for multiple reflection effects, the output from port 4 and port 5 will be almost in phase. The multiple reflection effects have been evaluated and cause, at most, a 0.4° phase error and 0.013 dB amplitude error for a

$K^2 = 2$ power divider at $\theta = 70°$. Thus, for most practical purposes, the split-tee power divider with single stage output transformers provides two equal phase outputs of constant amplitude ratio over a 1.6 to 1 band. The major effect of using single stage rather than multiple stage output transformers is a deterioration of the VSWR looking into the ports.

Experimental Results

In order to test the practicality of the theory, an L-band split-tee power divider with a two-third, one-third power ratio was constructed in strip line. The unit is shown in Fig. 8. Note that since junction effects do exist and line widths are not insignificant portions of a wavelength, the first design did not have equal electrical line lengths from the input to the isolation resistor. The first design was tested over a 15 per cent bandwidth and was found to have good amplitude and phase characteristics except that the signal from port 4 led that from port 5 by about 5°. To eliminate the 5° phase error and some mismatch effects, the modifications shown in dashed lines in Fig. 8 were made. The final unit has the characteristics shown in Figs. 9 and 10. Although the results are not equal to the theoretical results of the unequal output impedance divider, the output phase and amplitude is unusually independent of frequency.

A Modified Split-Tee Power Divider

The described split-tee power divider optimizes the phase equality transfer characteristic. However, the input VSWR is rather high at the band edges even for the unequal output impedance device. From an examination of (5) and Fig. 5, it is seen that the input VSWR is exactly the same as that obtained from a quarter-wave transformer. Thus, the input VSWR can be improved by using a two-step transformer at the input as shown in Fig. 11.[9] Over the considered band, this modification reduces the input VSWR to less than 1.09 to 1, improves the isolation to greater than 20 dB, and increases the output VSWR to about 1.2 to 1. Better input VSWR performance can be achieved if a Chebyshev-type input transformer is used.[10]

Conclusion

Design equations for a new, compact, easily constructed, power divider have been derived. Over a 1.6 to 1 band, this device provides two isolated outputs having equal phase and constant arbitrary power ratio. This unit is simpler and smaller than other power-divider designs having comparable performance.

[9] Brigham, E., Sylvania Electric Products suggested the modification.
[10] Cohn, S. B., Optimum design of stepped transmission line transformers, *IRE Trans. on Microwave Theory and Techniques*, vol MTT-4, Apr 1955, pp 16–21.

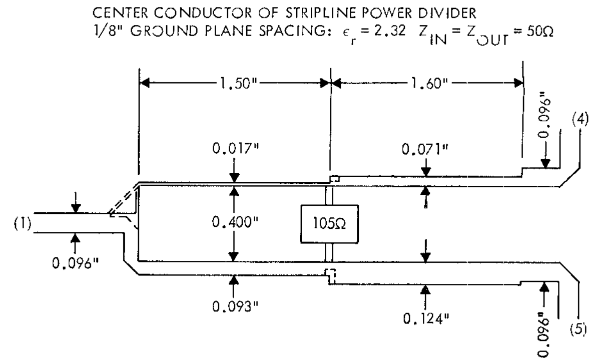

Fig. 8. Two-third, one-third power divider.

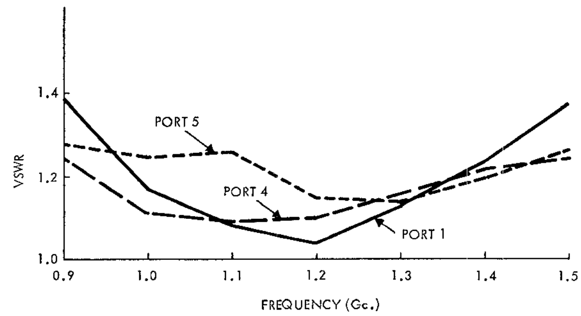

Fig. 9. Measured VSWR of split-tee power divider.

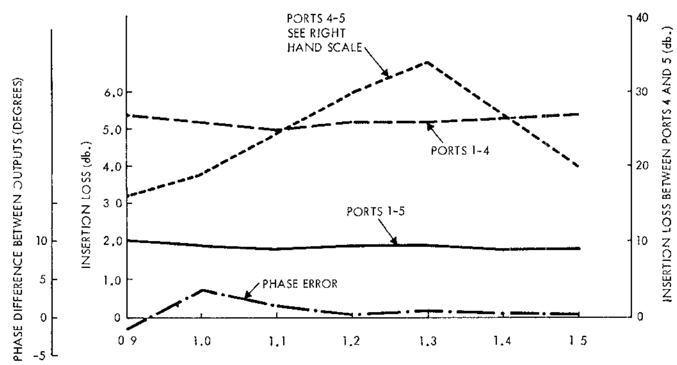

Fig. 10. Measured insertion loss and phase error.

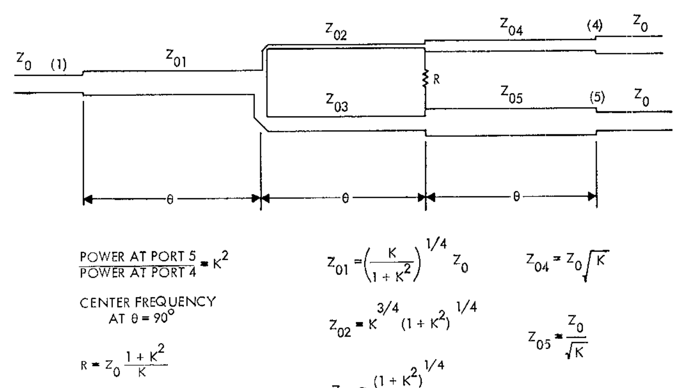

Fig. 11. Improved split-tee power divider.